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Heavy-quarkonium interaction in QCD at finite temperature

François Arleo^{a,b,1}, Joseph Cugnon^a, Yuri Kalinovsky^a

^a*Université de Liège, Institut de Physique B5
 Sart Tilman, 4000 Liège 1, Belgium*

^b*LPTHE, Université Paris VI & Paris VII and CNRS,
 4, Place Jussieu, 75252 Paris cedex 05, France*

Abstract

We explore the temperature dependence of the heavy-quarkonium interaction based on the Bhanot - Peskin leading order perturbative QCD analysis. The Wilson coefficients are computed solving the Schrödinger equation in a screened Coulomb heavy-quark potential. The inverse Mellin transform of the Wilson coefficients then allows for the computation of the 1S and 2S heavy-quarkonium gluon and pion total cross section at finite screening/temperature. As a phenomenological illustration, the temperature dependence of the 1S charmonium thermal width is determined and compared to recent lattice QCD results.

Key words: Heavy-quarkonium; Finite temperature

1 Introduction

The Debye screening between two opposite color charges is clearly seen in the QCD static potential computed at finite temperature T on the lattice [1]. Consequently, heavy quark bound states (which we call Φ) may no longer exist well above the deconfinement critical temperature T_c , of order 200–300 MeV [2]. This has made the heavy-quarkonium suppression in high energy heavy-ion collisions (as compared to proton-proton scattering) one of the most popular signatures for quark-gluon plasma formation [3, 4]. On the experimental

¹ Corresponding author.

Email address: arleo@lpthe.jussieu.fr (François Arleo).

side, a lot of excitement came out a few years ago after the NA50 collaboration reported a so-called “anomalous” suppression in the J/ψ channel in the most central lead-lead collisions ($\sqrt{s} \simeq 17$ GeV) at the CERN SPS [5]. At RHIC energy ($\sqrt{s} = 200$ GeV), J/ψ production has been measured recently by the PHENIX collaboration although the presently too large statistical and systematic error bars prevent one from concluding anything yet quantitative from these data [6].

The NA50 measurements triggered an intense theoretical activity and subsequently a longstanding debate on the origin of the observed J/ψ suppression. However, it became unfortunately rapidly clear that no definite conclusion could be drawn as long as theoretical uncertainties exceed by far that of the high statistics data. Indeed, both the realistic description of the space-time evolution of the hot and dense medium as well as the interaction of heavy-quarkonia with the relevant degrees of freedom (let them be pions or gluons) are required to be known. While the former can be constrained by global observables, the latter needs to be computed theoretically. Several approaches have been suggested to determine heavy-quarkonium total cross sections, from meson exchange [7] or constituent quark models [8] to the perturbative framework developed by Bhanot and Peskin [9, 10] upon which the present paper relies. Let us remark in particular that many recent phenomenological applications have used the latter perturbative Φ – gluon cross section to estimate the heavy-quarkonium dissociation or formation in heavy-ion collisions [11].

However, although derived from first principles in QCD perturbation theory, the Bhanot - Peskin result describes the interaction of Coulombic bound states, that is for which the heavy-quark potential is well approximated by the perturbative one-gluon exchange potential. As indicated from spectroscopic studies [12], this may be too crude an assumption to describe bound states in the charm or (even) the bottom sector. Furthermore, it does not take into account the possible effects of the medium on the heavy-quarkonium interaction. It is the aim of this Letter to explore how the Φ interaction with gluons and pions gets modified at finite temperature. The paper is organized as follows. The general framework is first briefly recalled in Section 2. Our results are then detailed in Section 3 while Section 4 is devoted to a concluding discussion.

2 Heavy-quarkonium interaction in QCD

2.1 Resummation of the leading-twist forward scattering amplitude

At leading-twist, the forward heavy-quarkonium (Φ) - hadron (h) scattering amplitude $\mathcal{M}_{\Phi h}$ is an operator product expansion of perturbative Wilson coefficients d_{2k} evaluated in the heavy-quarkonium state and computable in

perturbation theory times non-perturbative matrix elements in the hadron state. It reads [9]

$$\mathcal{M}_{\Phi h}(\lambda) = \left(\frac{g^2 N_c}{16 \pi} \right) a_0^2 \sum_{k \geq 1} d_{2k} \epsilon^{1-2k} \langle h | \frac{1}{2} F^{0\nu} (iD^0)^{2k-2} F_\nu{}^0 | h \rangle \quad (1)$$

where a_0 and ϵ stand, respectively, for the Bohr radius and the binding energy for the Φ system, g the QCD coupling and N_c the number of colors. Each of the matrix elements $\langle h | \dots | h \rangle$ in Eq. (1) is proportional to a traceless fully symmetric rank $2k$ tensor in the spin-averaged hadron state [9]

$$\Pi^{\mu_1 \dots \mu_{2k}}(p) = p^{\mu_1} \dots p^{\mu_{2k}} - \text{trace terms}$$

where p^μ is the hadron momentum. The trace terms correspond to target mass corrections $\mathcal{O}(m_h^2/\epsilon^2)$ which are neglected here as we shall deal only with pions in the present approach. Note that such corrections were systematically included in Refs. [13, 14] and proved relevant only slightly above the threshold for the quarkonium-hadron interaction process. The matrix elements can be written as

$$\langle h | \frac{1}{2} F^{0\nu} (iD^0)^{2k-2} F_\nu{}^0 | h \rangle = A_{2k} \Pi^{0 \dots 0}(p) = A_{2k} \lambda^{2k} \quad (2)$$

where $\lambda \equiv p^0$ is the hadron energy in the Φ rest frame and the A_{2k} coefficients are the Mellin transform of the unpolarized gluon density G^h in the hadron target [10, 14]

$$A_{2k} = \int_0^1 \frac{dx}{x} x^{2k} G^h(x).$$

Plugging (2) in (1), the leading-twist forward scattering amplitude can be written as

$$\mathcal{M}_{\Phi h}(\lambda) = \left(\frac{g^2 N_c}{16 \pi} \right) a_0^2 \epsilon \sum_{k \geq 1} d_{2k} A_{2k} (\lambda/\epsilon)^{2k}. \quad (3)$$

Expressing the Wilson coefficients in terms of their Mellin moments,

$$d_{2k} = \int_0^1 \frac{dx}{x} x^{2k} \tilde{d}(x),$$

the power series (3) can be conveniently resummed and continued analytically throughout the whole complex plane of energies [14]. This allows for the computation of the imaginary part of the forward scattering amplitude along the real axis, $\lambda > \epsilon$,

$$\text{Im } \mathcal{M}(\lambda) = \left(\frac{g^2 N_c}{32} \right) a_0^2 \epsilon \int_{\epsilon/\lambda}^1 \frac{dx}{x} G(x) \tilde{d}\left(\frac{\epsilon}{\lambda x}\right). \quad (4)$$

Dividing Eq. (4) by the flux factor λ leads to the total heavy-quarkonium cross section via the optical theorem

$$\sigma_{\Phi h}(\lambda) = \frac{1}{\lambda} \text{Im } \mathcal{M}(\lambda) = \int_0^1 dx G(x) \sigma_{\Phi g}(x\lambda), \quad (5)$$

where the heavy-quarkonium gluon cross section is defined as

$$\sigma_{\Phi g}(\omega) = \left(\frac{g^2 N_c}{32} \right) a_0^2 \frac{\epsilon}{\omega} \tilde{d}\left(\frac{\epsilon}{\omega}\right) \quad (6)$$

with the gluon energy $\omega = \lambda x$ in the Φ rest frame.

The Wilson coefficients need first to be computed in an arbitrary heavy-quark potential and later be inverse Mellin transformed in order to determine the heavy-quarkonium gluon Eq. (6) and hence the heavy-quarkonium hadron Eq. (5) total cross sections. This task is carried out in the next Section.

2.2 Wilson coefficients and inverse Mellin transform

Resuming all diagrams contributing to leading order in g^2 to the $\Phi - h$ interaction, Peskin made explicit the heavy-quarkonium Wilson coefficients [9]. They are given by[†]

$$\begin{aligned} d_{2k} &= \frac{16\pi}{N_c^2 a_0^2} \epsilon^{2k-1} \langle \phi | r^i \frac{1}{(H_a + \epsilon)^{2k-1}} r^j | \phi \rangle \\ &= \frac{16\pi}{N_c^2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{3} \left| \frac{\mathbf{r}}{a_0} \psi(k) \right|^2 \epsilon^{2k-1} \langle k | \frac{1}{(H_a + \epsilon)^{2k-1}} | k \rangle \end{aligned} \quad (7)$$

where $|\phi\rangle$ and k are respectively the $Q\bar{Q}$ internal wavefunction and momenta, while H_s (H_a) is the internal Hamiltonian describing the heavy-quarkonium state in a color-singlet (color-adjoint) state,

$$H_{s,a} = \frac{k^2}{m_Q} + V_{s,a}(r),$$

m_Q being the heavy-quark mass and $V_{s,a}$ the heavy-quark potential. The heavy-quarkonium wave function $\psi(r)$ in coordinate space and the binding energy ϵ appearing in Eq. (7) are determined solving the Schrödinger equation

$$H_s |\phi\rangle = -\epsilon |\phi\rangle \quad (8)$$

in the color singlet potential.

[†] Note that the coefficients (7) are a factor $(\epsilon/\epsilon_0)^{2k-1}$ smaller than in Ref. [9]. This difference is because the energy λ is normalized to the binding energy ϵ in the amplitude (3) and not to the Rydberg energy ϵ_0 as in [9].

2.2.1 Coulomb potential

The leading-twist amplitude (1) was determined assuming the $Q\bar{Q}$ binding potential is well approximated by the one-gluon exchange Coulomb potential

$$\begin{aligned} V_s &= -\frac{g^2 N_c}{8\pi r} + \mathcal{O}(N_c^{-1}), \\ V_a &= \mathcal{O}(N_c^{-1}), \end{aligned} \tag{9}$$

in $SU(N_c)$ gauge theory. To leading order in $\mathcal{O}(N_c^{-1})$, H_a is given by the free-particle Hamiltonian and the Wilson coefficients (7) read

$$d_{2k} = \frac{16\pi}{N_c^2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{3} \left| \frac{\mathbf{r}}{a_0} \psi \right|^2(k) \frac{(\epsilon/\epsilon_0)^{2k-1}}{[(ka_0)^2 + \epsilon/\epsilon_0]^{2k-1}},$$

where we have introduced the Rydberg energy ϵ_0 for the $Q\bar{Q}$ system

$$\epsilon_0 = \left(\frac{g^2 N_c}{16\pi} \right)^2 m_Q = \frac{1}{m_Q a_0^2}$$

Solving the Schrödinger equation (8) gives the well-known 1S and 2S Coulomb wave functions with the corresponding binding energies,

$$\begin{aligned} a_0^{3/2} \psi^{(1S)}(r) &= \frac{1}{\sqrt{\pi}} \exp\left(-\frac{r}{a_0}\right) & ; & & \epsilon_{1S} &= \epsilon_0 \\ a_0^{3/2} \psi^{(2S)}(r) &= \frac{1}{\sqrt{8\pi}} \left(1 - \frac{r}{2a_0}\right) \exp\left(-\frac{r}{2a_0}\right) & ; & & \epsilon_{2S} &= \epsilon_0/4 \end{aligned}$$

which eventually allows for the computation of the Wilson coefficients [9]

$$\begin{aligned} d_n^{(1S)} &= \int_0^1 \frac{dx}{x} x^n \frac{16^3}{3N_c^2} x^{5/2} (1-x)^{3/2}, \\ d_n^{(2S)} &= \int_0^1 \frac{dx}{x} x^n \frac{16 \times 16^3}{3N_c^2} x^{5/2} (1-x)^{3/2} (1-3x)^2. \end{aligned} \tag{10}$$

From Eqs. (6) and (10), the expression for the inverse Mellin transform $\tilde{d}(x)$ is straightforward and one gets directly [10]

$$\sigma_{\Phi^{(1S)} g}(\omega) = \frac{16^2 g^2}{6N_c} a_0^2 \frac{(\omega/\epsilon_{1S} - 1)^{3/2}}{(\omega/\epsilon_{1S})^5} \theta(\omega - \epsilon_{1S}), \tag{11}$$

for 1S states and [14]

$$\sigma_{\Phi^{(2S)} g}(\omega) = 16 \times \frac{16^2 g^2}{6N_c} a_0^2 \frac{(\omega/\epsilon_{2S} - 1)^{3/2} (\omega/\epsilon_{2S} - 3)^2}{(\omega/\epsilon_{2S})^7} \theta(\omega - \epsilon_{2S}) \tag{12}$$

for 2S states. Note that these expressions were also obtained by Kim, Lee, Oh and Song from the QCD factorization property combined with the Bethe-Salpeter amplitude for the heavy-quark bound state, which allowed them to include relativistic and next-to-leading order corrections [15].

2.2.2 Screened Coulomb potential

As stressed in the Introduction, the above formulas may serve as an important input to estimate the heavy-quarkonium dissociation process $\Phi + g \rightarrow Q + \bar{Q}$ (or the detailed balance process) in a hot gluon or pion gas formed in high energy heavy-ion collisions. We would like here to go one step further and to discuss possible medium modifications to these total cross sections. Medium effects will be modeled at the level of the heavy-quarkonium potential by considering a screened Coulomb potential (Yukawa type) characterized by a dimensionless screening parameter μ ,

$$V_s = -\frac{g^2 N_c}{8\pi r} \exp(-\mu r/a_0), \quad (13)$$

$$V_a = 0.$$

Solving the Schrödinger equation (8) using the potential (13), the wave functions and binding energies for 1S and 2S states are determined and the corresponding Wilson coefficients (7) are computed numerically subsequently. For the illustration, we plot in Figure 1 the typical size (mean and root mean square radii, *top*) as well as the binding energy (*bottom*) for the 1S (*left*) and 2S (*right*) Φ states. Finally, the inverse Mellin transform

$$\tilde{d}(x) = \frac{1}{2i\pi} \int_{c-i\infty}^{c+i\infty} dz x^{-z} d(z)$$

c being a real constant, is performed thus giving access to the medium-modified total cross sections.

3 Results

3.1 Finite screening

Before discussing the results, both the Bohr radius a_0 and the Rydberg energy ϵ_0 in the charmonium and bottomonium channel need to be fixed. Assuming both the 1S and 2S states to be Coulombic, the heavy quark mass m_Q and the Rydberg energy ϵ_0 can be determined from the 1S and 2S heavy-quarkonium masses. One then obtains [14]

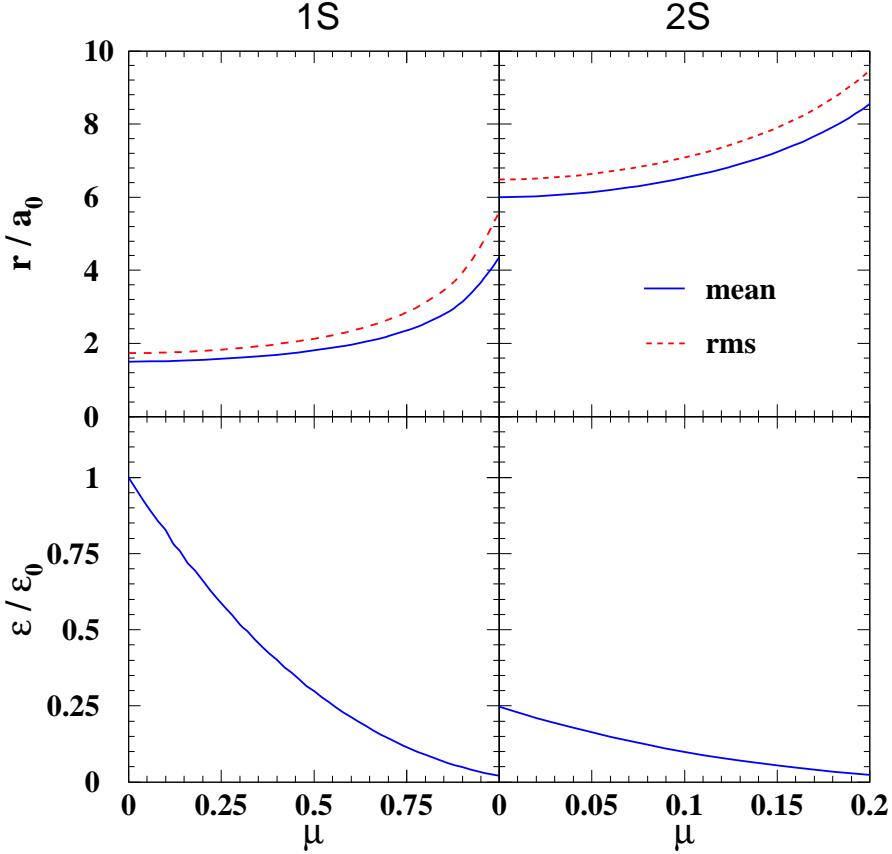


Fig. 1. Top: Mean radius (solid) and root mean square radius (dashed) of the 1S (left) and 2S (right) heavy-quarkonium states as a function of μ . Bottom: 1S (left) and 2S (right) binding energy as a function of μ .

$$\begin{aligned}\epsilon_{0c} &= 0.78 \text{ GeV}, \quad a_{0c}^{-1} = 1.23 \text{ GeV}, \\ \epsilon_{0b} &= 0.75 \text{ GeV}, \quad a_{0b}^{-1} = 1.96 \text{ GeV}.\end{aligned}$$

Using the above (not too hard) scales, the 1S (*top*) and 2S (*bottom*) heavy-quarkonium gluon dissociation cross sections are computed in Figure 2 as a function of the gluon energy ω for various values of the screening parameter μ . The dominant effect of the screened heavy quark potential is the decrease of the 1S (respectively, 2S) heavy-quarkonium binding energy from ϵ_0 (respectively, $\epsilon_0/4$) to ϵ which leads to a lower threshold for the inelastic process. The medium modifications of the Φ – gluon total cross sections are nevertheless not only due to the smaller binding energy, yet the characteristic shapes of the cross sections are reminiscent to what is already known for pure Coulombic states, $\mu = 0$ (Figure 2, *solid*). We checked for instance that the Wilson coefficients get somehow modified at finite screening and consequently the partonic cross sections do not simply scale as ω/ϵ in Eq. (11). This is a strong indication that cross sections cannot be deduced with a simple rescaling of the binding energy from ϵ_0 to ϵ to mimic medium effects in the heavy-quarkonium

dissociation process. Finally, the significant increase of the 1S partonic cross sections at large screening is particularly noticeable as the dipole size gets larger. However, as discussed later, reliable calculations require the space-time scales to remain small which prevent one from taking arbitrarily large screening parameter values, at least when considering such “light” heavy quarks[‡]. Moreover, since the heavy-quark potential in the original QCD analysis needs to be Coulomb-like, the screening parameter μ in the model Eq. (13) should remain small as compared to one.

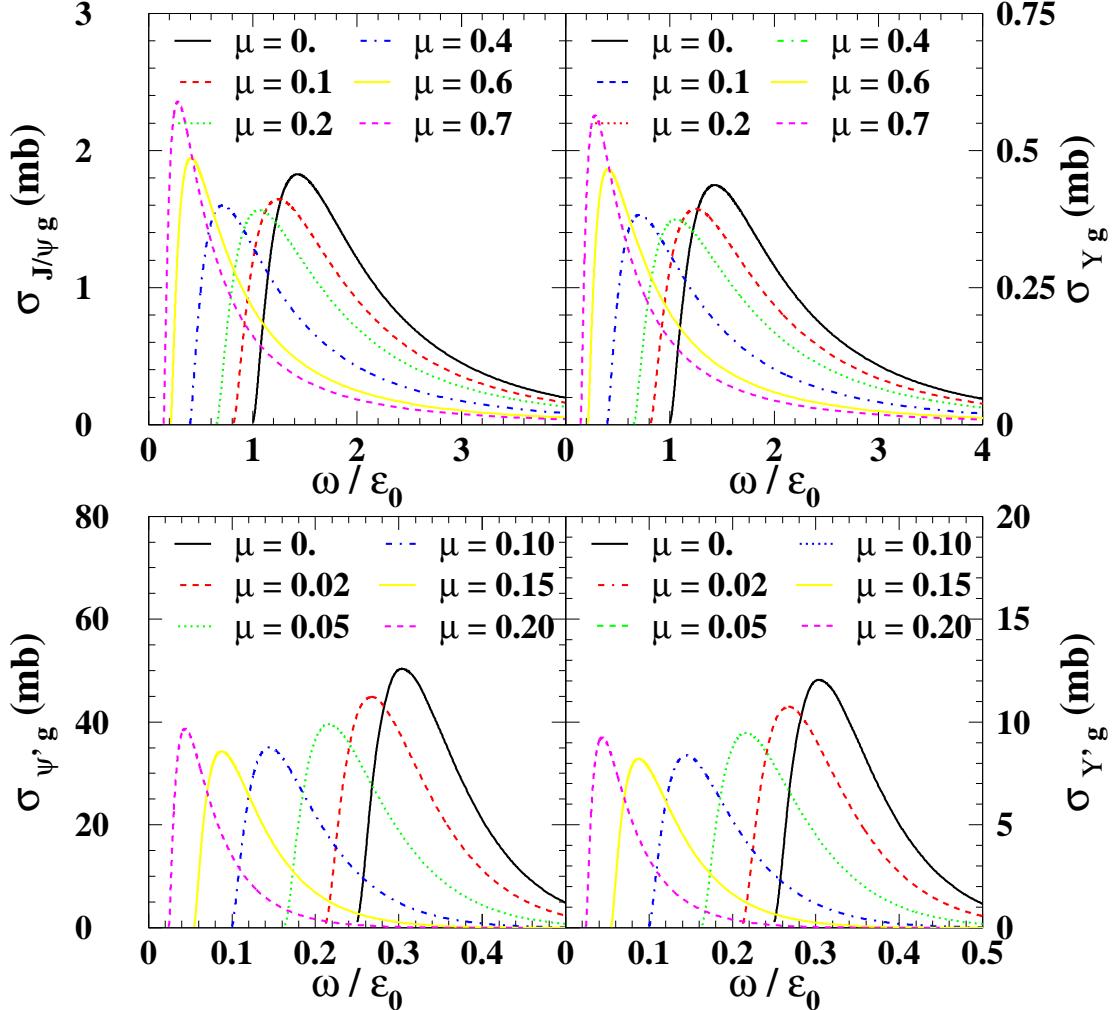


Fig. 2. 1S (*top*) and 2S (*bottom*) charmonium (*left*) and bottomonium (*right*) gluon total cross section as a function of the gluon energy ω for various values of the screening parameter μ .

Let us now discuss the heavy-quarkonium hadron cross section. Since heavy-quarkonia plunged into the hot medium are most likely to interact with pions,

[‡] According to [10], the assumption of heavy-quark Coulombic bound states should be appropriate for more than 25 GeV heavy quark mass.

we shall only consider the $\Phi - \pi$ channel and choose the GRV LO parameterization for the gluon distribution[§] in the pion [16]. The $J/\psi - \pi$ and $\Upsilon - \pi$ cross sections are computed in Figure 3 as a function of the pion energy λ . Again, the threshold for the process, located at $\lambda = \epsilon$, gets shifted to lower values leading to a strong modification of the heavy-quarkonium pion interaction in this region. At high energy, small $x = \mathcal{O}(\epsilon/\lambda)$ gluons dissociate heavy-quarkonia, thereby increasing the $\Phi - \pi$ cross section by a factor $(\epsilon_0/\epsilon)^\delta$ where $\delta \simeq 0.3$ governs the rise of the gluon distribution at small x , $xG(x) \propto x^{-\delta}$ [17].

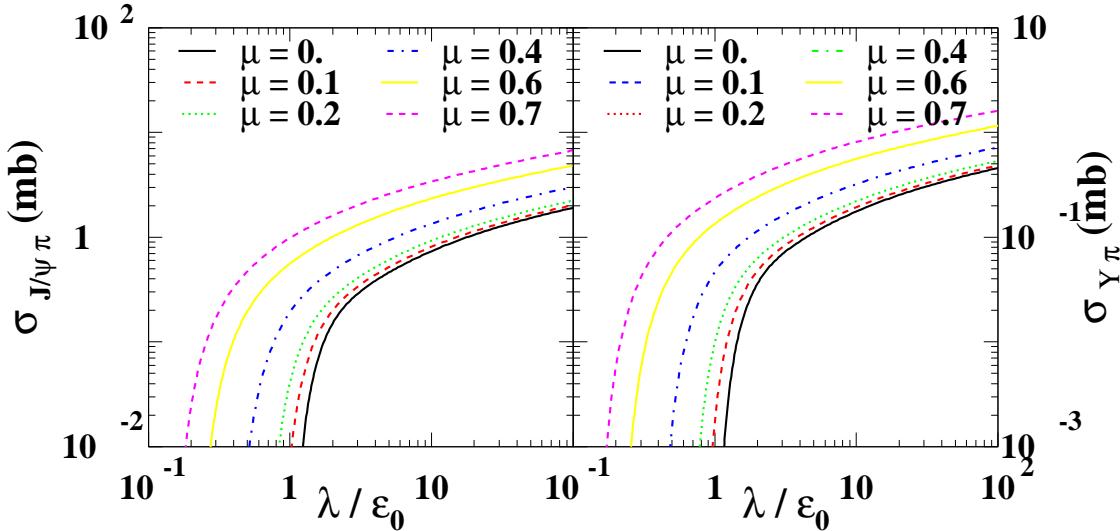


Fig. 3. $J/\psi - \pi$ (left) and $\Upsilon - \pi$ (right) total cross section as a function of the pion energy λ for various values of the screening parameter μ .

3.2 Finite temperature

The Φ interaction with gluons and pions has been computed so far using a heavy-quark screened Coulomb potential characterized by one parameter μ . Interpreting μ as the screening mass in a gluon plasma, the model for the finite temperature $Q\bar{Q}$ potential now looks like

$$V_s = -\frac{g^2(r, T) N_c}{8\pi r} \exp(-m_D(T)r) \quad (14)$$

At short distance and/or low temperature, we shall consider a frozen coupling constant

$$g^2(r, T) = g^2 \quad \text{for } rT \ll \Lambda \quad (15)$$

[§] These should be evaluated at a factorization scale ϵ . We take in the following a frozen scale ϵ_0 .

and recover the Coulomb potential behavior (9), while the QCD coupling starts to run with T at large distance and/or high temperature. At two loops, we have

$$g^2(r, T) \equiv \tilde{g}^2(T) = \left(\frac{11}{8\pi^2} \ln \left(\frac{2\pi T}{\Lambda_{\overline{\text{MS}}}} \right) + \frac{51}{88\pi^2} \ln \left[2 \ln \left(\frac{2\pi T}{\Lambda_{\overline{\text{MS}}}} \right) \right] \right)^{-1} \quad \text{for } r T \gg \Lambda \quad (16)$$

with $T_c/\Lambda_{\overline{\text{MS}}} = 1.14$ [18]. The Debye mass m_D is related to the temperature through the leading-order perturbative result,

$$m_D(T) = \tilde{g}(T) T.$$

The Λ dimensionless parameter introduced in Ref. [18] separates somewhat arbitrarily the short from the long distance physics at finite temperature. Fitting pure gauge SU(3) heavy quark potential, they obtained the empirical value $\Lambda = 0.48 \text{ fm} \times T_c$. Following [18], we shall take the 2-loop running coupling (16) rescaled by 2.095 and interpolate smoothly between the short and long distance regime[¶].

The partonic and hadronic J/ψ and Υ cross sections are computed in Figure 4 for several temperatures in units of the critical temperature for deconfinement, $T_c = 270 \text{ MeV}$ in SU(3) pure gauge theory [2]. The temperatures selected for the bottomonium system are chosen to be slightly higher than those for the charmonium system since the larger bottom quark mass (hence, smaller size) probes more efficiently hotter QCD media [19].

The effects of the running coupling in Eq. (14) being quite small, rather similar features at finite temperature and at finite screening are observed. In particular, the charmonium binding energy (hence the inelastic threshold) drops by a factor of two already at $T/T_c = 0.5$ and thus affects dramatically the J/ψ interaction in the vicinity of the threshold. At higher temperature, the J/ψ – gluon cross section is significantly enhanced at small gluon energy due to the larger charmonium size. The J/ψ – π cross section is also somewhat modified with a magnitude increasing noticeably with the temperature. Moving to the bottom sector (Figure 4, right), the Υ cross sections exhibit the same general characteristics yet the medium effects at a given temperature prove much less pronounced from the smaller bottomonium size.

At high temperature, heavy-quarkonium interaction can not be described by short-distance techniques (see Fig. 1) and our predictions are not valid any longer. On top of that, the process described here is the heavy-quarkonium dissociation by *hard* gluons as opposed to the soft gluons which only affect its properties. Therefore, our calculations should be valid as long as

[¶] Similar results are obtained using the one loop running coupling with an appropriate rescaling.

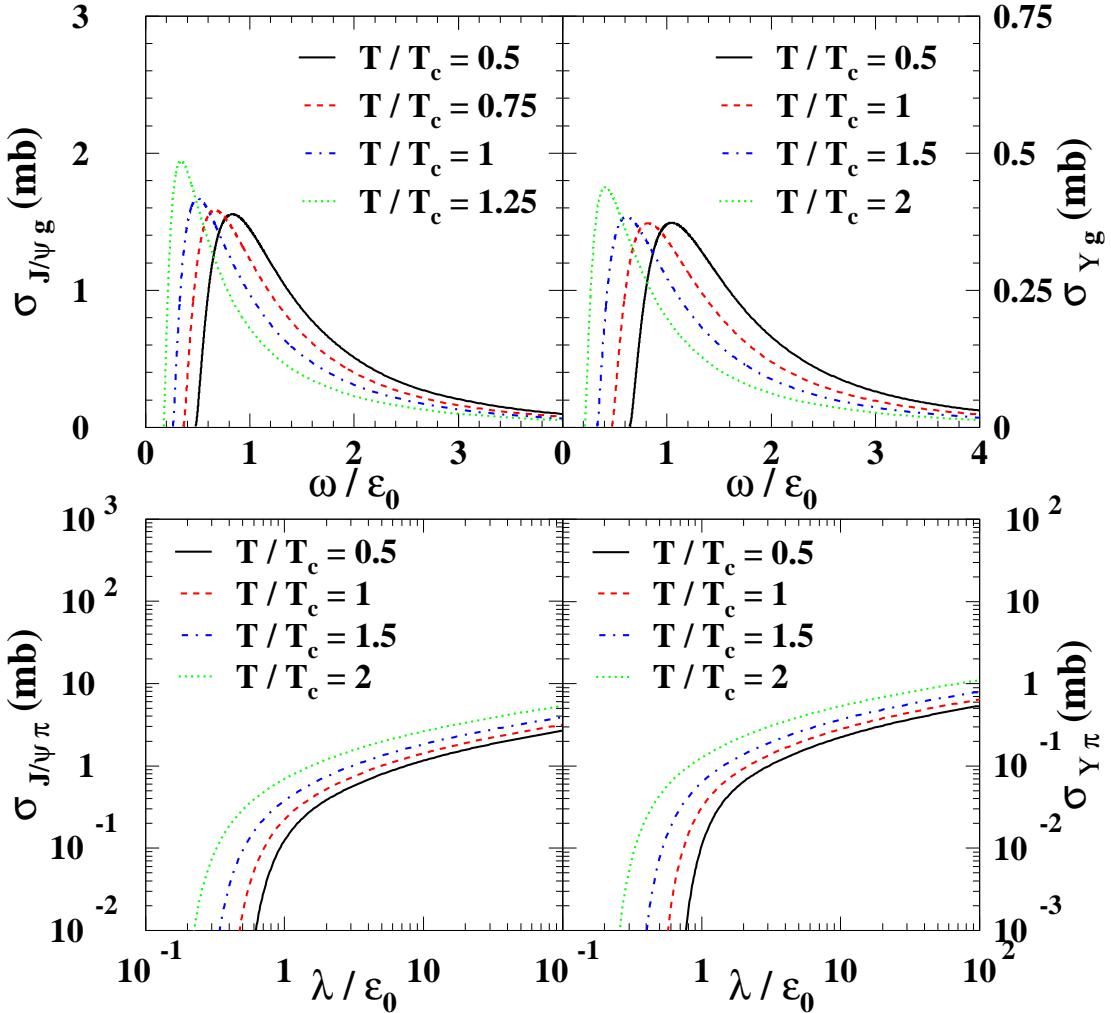


Fig. 4. J/ψ (left) and Υ (right) total cross sections with gluons (top) and pions (bottom) at various temperatures.

the Debye mass is kept smaller than the heavy-quarkonium Rydberg energy, $m_D(T) \lesssim \varepsilon_0$. This condition is fulfilled provided the bath temperature is smaller than 350 MeV. Above that scale, the screened exchanges are able to dissociate the bound states, the factorization between the heavy-quarkonium physics and the external gluon field is broken and the above QCD picture loses its significance.

3.3 J/ψ spectral function width

The former results indicate that Debye screening effects may play an important role in the heavy-quarkonium dissociation by incoming gluons or pions. In order to illustrate how medium modifications could affect the Φ suppression in heavy-ion collisions, we compute in this section the 1S charmonium thermal

width $\Gamma_{J/\psi}$ (or equivalently its lifetime, $\tau_{J/\psi} = \Gamma_{J/\psi}^{-1}$) in a hot gluon bath. Assuming the J/ψ suppression is only due to the gluon dissociation process, the width can be written

$$\Gamma_{J/\psi}(T) = \frac{1}{2\pi^2} \int_0^\infty \omega^2 d\omega \sigma_{J/\psi g}(\omega, T) n_g(\omega, T)$$

where $n_g(\omega, T) = 2(N_c^2 - 1)/(\exp(\omega/T) - 1)$ is the gluon density in a gluon gas in thermal equilibrium.

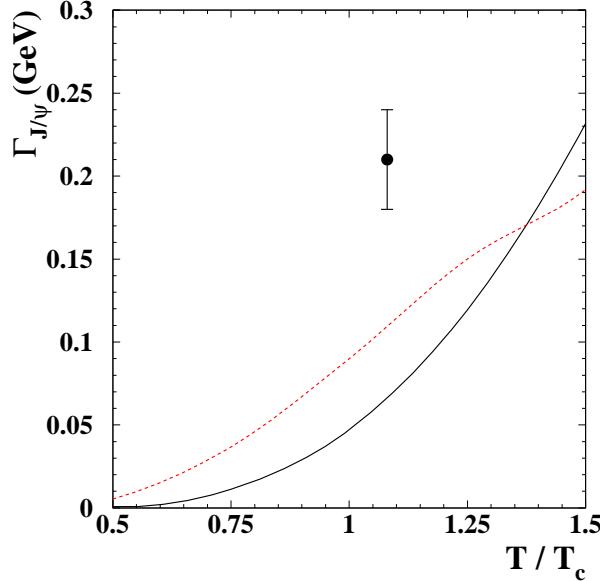


Fig. 5. J/ψ thermal width as a function of the temperature with (*dotted*) and without (*solid*) modifications of the heavy-quark potential. The lattice data point obtained in Ref. [20] is also shown for comparison.

The thermal width is computed in Figure 5 as a function of the temperature T assuming the vacuum (*solid*) and the in-medium (*dashed*) J/ψ – gluon cross section. At small temperature, $T \ll \epsilon$, most gluons are not sufficiently energetic to dissociate J/ψ states and the width remain small as the phase space selected by the J/ψ gluon threshold is restricted. When the medium gets warmer, more and more gluons are able to interact inelastically with the J/ψ , hence the thermal width increases. Interestingly enough, the in-medium J/ψ thermal width proves larger by a factor of two or more up to $T = T_c$ due to the lower threshold in the medium modified cross sections. At even higher temperature, the medium modified result becomes smaller to that in the vacuum since *dissociating* gluons (with ω of order ϵ) grow scarce. Also plotted in Figure 5 is the J/ψ width computed recently on the lattice at finite temperature in the quenched approximation [20]. Although a significant discrepancy remains between our calculations and the lattice data point, it is interesting to note that adding medium effects tends to reduce the disagreement, whose origin is not clarified.

4 Concluding discussion

Before summarizing our main results, we would like to discuss the limitations of our approach. The starting point of the calculation is the forward scattering amplitude $\mathcal{M}_{\Phi h}$ originally derived for Coulomb bound states. To go beyond this one-gluon exchange picture would require to include light quark loops in the diagrammatics, to which the soft gluon source may couple, that we have not attempted. However, as conjectured in [10], it is appealing to guess that the generic dipole coupling appearing to leading order in g^2 in the heavy-quarkonium Wilson coefficients (7) survives perturbative and non-perturbative modifications of the $Q\bar{Q}$ binding potential. Therefore we believe that taking the literal expression for the Coulomb states Wilson coefficients and compute them in a screened Coulomb potential appears sensible, at least as long as the screening remains reasonable, $m_D a_0 \ll 1$. This is certainly the case when the temperature is kept small as compared to the heavy quark mass. In that sense, the smallness of the charm and bottom quark mass as compared to the non-perturbative scale of QCD indeed remains a problematic issue. As we have seen, typical space time scale becomes increasingly larger with the temperature, thus strongly limiting our confidence in the high temperature regime. Finally, one should keep a clear factorization between the gluon source and the heavy-quarkonium swimming in the gluon bath. We have seen that such a separation should be achieved as long as the Debye mass is small as compared to the bound state Rydberg energy, that is for temperatures $T \lesssim 350$ MeV.

We presented a numerical calculation of the heavy-quarkonium cross section with gluons and pions, taking into account the possible medium-modifications of the heavy-quark potential at finite temperature. Such a work can therefore be useful to estimate heavy-quarkonium production in high energy heavy-ion collisions. In particular, we feel it would be interesting to explore the phenomenological consequences of such corrections comparing them to present calculations based on the vacuum heavy-quarkonium interaction. Finally, this very framework could be applied to study the Φ interaction using a variety of realistic heavy quark (confining) potentials currently used in charmonium and bottomonium spectroscopy [12] to describe more accurately, although further away from the perturbative requirement, heavy-quarkonium interaction with gluons and hadrons.

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